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Modelling the Conditional Variance of Returns on Agricultural Commodity Futures Contracts

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Abstract

In this paper various GARCH family models are examined to find the one best suited for modeling the conditional volatility of returns of soft commodity futures. The data came from the Chicago Mercantile Exchange (CME) and covered the period from 1987 to 2022. Three different commodities were selected, namely corn, soybeans and wheat. The most striking finding of this study is that there is not much difference between the models of conditional volatility for a given commodity and across the commodities under consideration. This demonstrates possible difficulties that one could face while trying to choose out of the competing parameterizations.

Keywords: agricultural products, estimation, conditional variance, GARCH model

1. Introduction

One of the characteristics of financial markets most closely observed by investors is volatility. The volatility of financial instrument prices is both a source of risk and a manifestation of the pervasive uncertainty about the course of economic processes and phenomena. On the other hand, periods of high price volatility in financial markets offer many investors the opportunity to earn small fortunes, provided they have successfully identified risk "hotspots" early on. It is, therefore, not surprising that the issue of volatility, and in particular the problem of modeling the variance of financial instrument return rates, has received considerable attention in the literature. The basic properties of the variance of the return rates of financial instruments are well-documented. A common phenomenon is the clustering of variance, where both large and small price changes of financial instruments tend to occur in series, exemplifying one of these properties. The distributions of return rates of financial instruments are also characterized by the presence of so-called heavy tails [see S. Deng, W. Jiang, Z. Xia, 2002; H.J. Jin, 2007; G.

Malik, 2011; 2013; T. Zhang at al., 2023]. A practical consequence of the aforementioned phenomenon is a higher probability of atypical observations (large increases or large decreases) compared to the normal distribution. Variance is also negatively correlated with changes in financial instrument prices. During market downturns, when prices generally fall, relative changes will be greater if absolute changes remain on average the same. This phenomenon is related to the so-called leverage effect, which typically has a greater impact of negative return rates on the conditional variance of the series.

In this paper, an analysis was conducted to assess the suitability of three popular GARCH class models, namely GARCH(S, W), EGARCH(S, W), and GJR(S, W), for modeling the conditional variance of return rates of futures contracts on corn, wheat, and soybeans. The data came from the Chicago Mercantile Exchange and covered the years from 1987 to 2022. The empirical analysis demonstrated that the modeled series exhibit a high degree of similarity. This is reflected in the similar values of the parameters of the competing models, as well as the values of other characteristics, such as the value of the popular AIC criterion.

The article is divided into five parts. The second chapter includes a description of the research sample and basic descriptive statistics of the series used in the study. The third chapter is devoted to discussing the methodology of modeling the conditional variance of return rate series, presenting the models, and describing the subsequent stages of the research. Empirical results are presented in the fourth chapter, while the fifth and final chapter contains conclusions.

2. Description of the Research Sample

The data used for analysis in the empirical part of the study were sourced from the Chicago Mercantile Exchange (CME). "The research sample consists of daily quotes of nominal prices of futures contracts for three agricultural products: corn, wheat, and soybeans. The contract value is expressed as the price per bushel of the respective commodity unit in US dollars. The data include the closing price of contracts with the shortest expiration term, allowing the series of quotations to be treated as futures prices with the shortest possible realization term. The selection of products was justified by their significance in the futures market and the availability of sufficiently long time series. The empirical data cover the years 1987-2022, and each individual time series comprises 9,090 observations. These data were checked for potential discontinuities and errors. To minimize the impact of arbitrary interventions on the obtained results, no procedures for data correction or supplementation were applied."

A detailed analysis of the price behavior of the studied agricultural products during the discussed period, along with graphical illustrations, basic descriptive statistics for the returns of the studied agricultural products, and the results of normality testing using the Shapiro-Wilk and Jarque-Bera tests can be found in the article. G. Malik [see G. Malik, 2024, January].

3. Methodology

Return series of financial instrument prices most often belong to the group of stationary processes or non-stationary processes whose degree of integration does not exceed one. Usually, there is also autocorrelation, although it rapidly diminishes for higher lags [see J. Brzeszczyński, R. Kelm, 2002; A. Weron, E. Weron, 2022]. The ARIMA model is suitable for series with a certain finite and integer degree of integration d and a dependency structure that includes both autoregressive parameters and moving average parameters of errors. In general, the ARIMA model is expressed by the formula [see G. Box, G. Jenkins 1983; 2013]:

$$\left(1-\sum_{k=1}^{p}\phi_k L^k\right)(1-L)^d r_t = \left(1-\sum_{k}^{q}\theta_k L^k\right)\varepsilon_t \tag{1}$$

where L denotes the lag operator.

The classical ARIMA model assumes a constant variance of the random error term ε_t over time. In any practical application of the ARIMA model, particularly for modeling time series returns of financial instruments, this assumption is usually not met. Therefore, it is necessary to extend model (1) by allowing for changes in the variance of the random error term over time, $\varepsilon_t \sim (0, h_t)$.

The variability of variance over time is described by a separate equation, the form of which allows differentiation between various groups of models. Although the presentation of GARCH models should ideally begin with the simple ARCH model, introduced by Engle [see R.F. Engle, 1982; 1990], this discussion will not lose its generality if the first model presented is the GARCH model, which is a generalization of the aforementioned model [see D.A. Dickey, W.A. Fuller, 1979]. The GARCH(S, W) model is expressed by the following equation:

$$h_t = \gamma_0 + \sum_{s=1}^{S}\gamma_s\varepsilon_{t-s}^2 + \sum_{w=1}^{W}\beta_w h_{t-w} \tag{2}$$

In the model given by equation (2), the conditional variance is explained by the lagged squared residuals and the lagged variance. The ARCH model does not include the autoregressive term and can therefore be considered a special case of the GARCH model, namely: GARCH(S, 0).

However, model (2) is not without certain drawbacks. In particular, in this model, the conditional variance does not depend on the sign of ε. It is well-known that there is a common phenomenon where prices react more strongly, and thus exhibit higher variance, to negative information entering the market. Additionally, the GARCH model does not allow for oscillatory behavior of disturbances. Not every "explosion" of variance that we may encounter in practice will have a decaying nature.

The EGARCH model [see D. Nelson, 1991] addresses these drawbacks. In general, the GARCH(S, W) model can be expressed as follows:

$$logh_t = \gamma_0 + \sum_{s=1}^{S}\left[\gamma_s\theta_{t-s} + \delta_s\left(|\theta_{t-s}| - \sqrt{\frac{2}{\pi}}\right)\right] + \sum_{w=1}^{W}\beta_w logh_{t-w} \tag{3}$$

where $\theta_t = \frac{\varepsilon_t}{\sqrt{h_t}}$.

The asymmetry in the response of variance to negative and positive residuals can also be expressed in the manner proposed by Golsten and others [see L.R. Golsten et al., 1983]. The GJR(S, W) model proposed by these authors is expressed by the following equation:

$$h_t = \gamma_0 + \sum_{s=1}^{S}[\gamma_s\varepsilon_{t-s}^2 + \delta_s I(\varepsilon_{t-s} \leq 0)\varepsilon_{t-s}^2] + \sum_{w=1}^{W}\beta_w logh_{t-w} \tag{4}$$

Among the many possible parameterizations of the conditional variance equation (values of S and W), the most commonly encountered in practice is where both S and W are equal to one. Hence, this parameterization has been adopted in the present study. The original intention that the order of lags in the conditional variance equations should be invariant to the models was also not insignificant.

The estimation of conditional variance models was preceded by determining the degree of integration of the series d and identifying the appropriate form of the mean equation, i.e., specifying the number of autoregressive (p) and moving average (q) parameters in formula (1). To examine the degree of integration of the prices of the considered agricultural products, the generalized DF test was applied [see W. W. Charemza, D. F. Deadman, 1997, pp. 114-117; D. A. Dickey, W. A. Fuller, 1979; D. A. Dickey, W. A. Fuller, 1981]. The number of lags in the test was empirically adjusted. The selection procedure involved starting the testing with the maximum lag, and then in subsequent rounds of the test, the lag was reduced by one until the null hypothesis of the existence of a unit root was rejected. Testing allowed for the determination of the degree of integration of the return series of agricultural commodity futures contracts at $d = 0$, which indicates stationarity. The selection of the specific model parameterization proceeded in two stages. General guidelines regarding the number of autoregressive and moving average parameters were provided by the behavior of the autocorrelation function and the partial autocorrelation function [see G. Box, G. Jenkins, 1983, p. 93]. The final test was the value of the Akaike information criterion.

4. Estimation Results of Selected Conditional Variance Models

The estimation results of the models considered are presented in Tables 2-4. In addition to the values of the parameters and their errors, these tables also include the value of the log likelihood function (LLF) and the Akaike Information Criterion (AIC). For comparison purposes, the values of the Bayesian Information Criterion (BIC), the Shibata Criterion (SC) and the Hannan-Quinn Criterion (HQC) are also given. To save space, the parameters of the mean equations have not been included in the tables. Full results are available upon request from the reader.

Table 1. GARCH(1,1) Model Estimation Results

	Corn	**Soybeans**	**Wheat**
$\gamma_0 10^3$	0,005 (0,001)	0,005 (0,001)	0,008 (0,001)
γ_1	0,095 (0,006)	0,083 (0,007)	0,079 (0,004)
β	0,905 (0,008)	0,915 (0,006)	0,912 (0,005)
LLF	25578,44	25755,37	23685,37
AIC	-5,644	-5,676	-5,223
BIC	-5,641	-5,675	-5,229
SC	-5,644	-5,674	-5,229
HQC	-5,643	-5,671	-5,226

Source: [own work]

Table 2. EGARCH(1,1) Model Estimation Results

	Corn	**Soybeans**	**Wheat**
γ_0	-0,145 (0,005)	-0,116 (0,016)	-0,103 (0,031)
γ_1	0,021 (0,007)	0,026 (0,006)	-0,007 (0,005)
δ	0,193 (0,009)	0,174 (0,007)	0,154 (0,021)
β	0,891 (0,003)	0,912 (0,002)	0,932 (0,003)
LLF	25555,58	25855,12	23795,42
AIC	-5,614	-5,673	-5,298
BIC	-5,662	-5,612	-5,293
SC	-5,714	-5,736	-5,299
HQC	-5,701	-5,712	-5,297

Source: [own work]

Table 3. GJR(1,1) Model Estimation Results

	Corn	Soybeans	Wheat
$\gamma_0 10^3$	0,005 (0,001)	0,004 (0,002)	0,008 (0,001)
γ_1	0,097 (0,007)	0,093 (0,007)	0,071 (0,006)
δ	-0,015 (0,008)	-0,033 (0,007)	0,014 (0,009)
β	0,907 (0,006)	0,919 (0,006)	0,912 (0,005)
LLF	25553,52	25655,15	23659,42
AIC	-5,643	-5,673	-5,232
BIC	-5,626	-5,621	-5,238
SC	-5,642	-5,673	-5,237
HQC	-5,617	-5,621	-5,236

Source: [own work]

A characteristic feature of the results is that both the log-likelihood values and the AIC values remain similar, not only across different models for the same product, where we observe instances of identical values, but also across different products. This observation has at least two important practical implications.

Firstly, the similarity in the aforementioned values highlights how functionally similar the agricultural products considered in the study are. Corn, soybeans, and wheat can be regarded as close substitutes. Therefore, it is somewhat understandable that the behavior and properties of futures contracts for these products are very similar.

All estimated parameters of the selected conditional variance models presented in Tables 1-3 were found to be statistically significant. The differences in parameter values for the same models across the agricultural products considered here are negligible. If we accept that the similarity of time series can be measured by the similarity of the models used to describe them, this observation leads to the conclusion that the examined series are indeed very similar.

It is also noteworthy that the parameter responsible for modeling the asymmetry in the variance response to positive and negative returns is negative. This result suggests an inversely proportional relationship between the squared lagged residuals and the level of variance. Therefore, declines in the futures market are accompanied by lower variance. A probable reason for this is the existence of "natural" support for products such as corn, soybeans, and wheat, which results from the irreducible and largely predictable demand from the livestock and fuel sectors.

When it comes to choosing between different conditional variance models for individual products, the problem is not an easy one. The small differences in AIC values render the Akaike criterion an ineffective tool. Additionally, testing the autocorrelation of standardized residuals and the squares of standardized residuals did not clearly indicate the superiority of any of the models considered here. Therefore, it seems most appropriate to refer to the fact that the EGARCH and GJR models allow for the modeling of a broader range of variance-related phenomena, particularly the asymmetry effects in the conditional variance equation. Hence, these models should be recommended for practical applications.

5. Conclusion

This paper examines the suitability of selected GARCH class models to describe the conditional variance of returns on agricultural futures contracts. The sample included daily values of corn, soybean and wheat contracts, covering the period from 1987 to 2022.

The study found a very strong similarity among the modeled series in terms of their properties. The estimated models do not differ significantly from each other across different products. Within a single product, it is difficult to identify a clear advantage for any of the considered parameterizations. The AIC values were very similar, and testing the autocorrelation of standardized residuals also did not reveal a dominant model.

Given the inconclusiveness of comparing different models for a given product, it seems reasonable to suggest that choices in this regard should be theory-based. This means selecting, for practical applications, the conditional variance model whose specification allows for the consideration of the broadest possible range of phenomena encountered in the practice of modeling the variance of financial instrument returns.

It should be noted, however, that the obtained results were based on a very large sample (each series comprised over 9,000 observations). In smaller samples, the AIC value may still play an important role in the model specification stage. Additionally, the estimation used the normal distribution as the conditional distribution. It is possible that the use of other conditional distributions may lead to a better explanation of the specific characteristics of the modeled series. This issue will be the focus of the author's future research.

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